3. Sound-Source Localization

1. Time difference of arrival (TDOA)
2. Estimation of TDOA by cross-correlation
3. Estimation of TDOA in the spectral domain
4. **The geometry of two microphones**
5. Direction of arrival
6. Using more than two microphones
7. Embedding the microphones in a robot head
8. Learning a sound propagation model
9. Predicting direction of a sound with a robot head
10. Example of sound direction estimation

Finding the Position of the Sound Source

The position of the sound source is given by solving the equation:

$$\hat{\tau} = \tau(\boldsymbol{S})$$

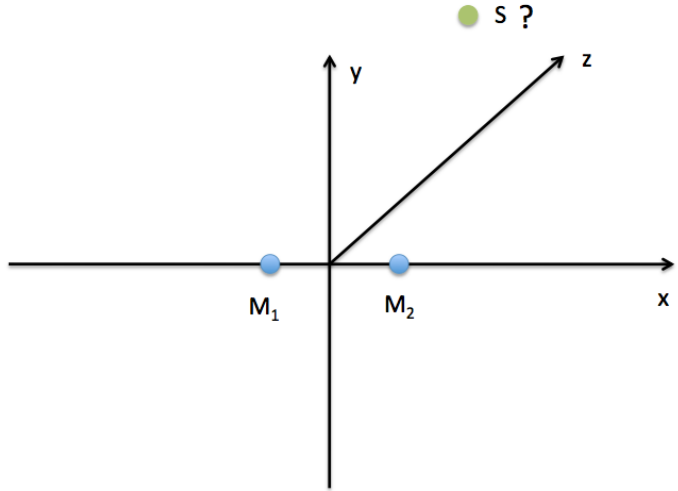
In more detail:

$$\hat{\tau} = \frac{\|\boldsymbol{S} - \boldsymbol{M}_1\| - \|\boldsymbol{S} - \boldsymbol{M}_2\|}{\nu}$$

Let $M_1 = (-m, 0, 0)$ and $M_2 = (m, 0, 0)$.

$S = (x, y, z)$ is the unknown source location.

Finding the Position of the Sound Source



Finding the Position of the Sound Source

The equation develops as:

$$\nu\hat{\tau} = \left((x+m)^2 + y^2 + z^2\right)^{1/2} - \left((x-m)^2 + y^2 + z^2\right)^{1/2}$$

which be written as:

$$\frac{x^2}{\frac{\nu^2\hat{\tau}^2}{4}} - \frac{y^2}{\frac{4m^2-\nu^2\hat{\tau}^2}{4}} - \frac{z^2}{\frac{4m^2-\nu^2\hat{\tau}^2}{4}} = 1$$

Two-Sheet Hyperboloid $\mathcal{H}$

This equation has the following structure:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{b^2} = 1$$

This is a *two-sheet hyperboloid*, $\mathcal{H}$, with foci in $\boldsymbol{M}_1$ and $\boldsymbol{M}_2$.

The sound source $S = (x, y, z)$ should lie on $\mathcal{H}$.

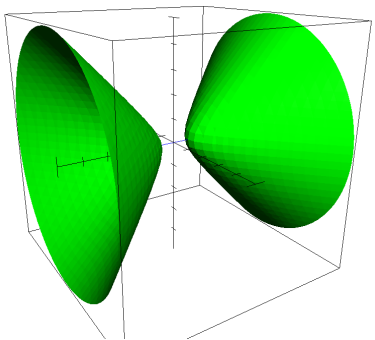
Where is the Sound Source?

An estimated time difference of arrival $\hat{\tau}$ defines a surface $\mathcal{H}_{\hat{\tau}}$.

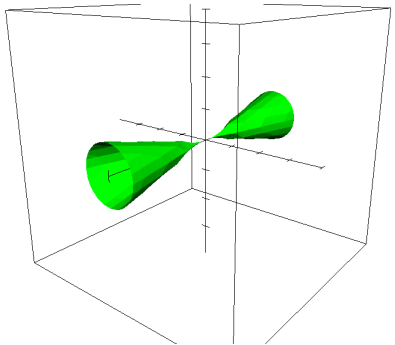
The surface exists only if:

$$0 < |\hat{\tau}| < \frac{\|M_1 - M_2\|}{\nu}$$

Two-Sheet Hyperboloids



small TDOA



large TDOA

Choosing a Sheet

- The hyperboloid has to sheets (branches), one with $x < 0$ and one with $x > 0$.
- The source can be on either one of these two branches. From the above equations, we can write:

$$x = \frac{\nu\hat{\tau}\,(2\|\boldsymbol{S} - \boldsymbol{M}_1\| - \nu\hat{\tau})}{4m}$$

The sign of x, that depends on $\hat{\tau}$, selects one of the two sheets.

Session Summary

- The TDOA is combined with the geometry of two microphone and an emitting source
- For any two given microphones, the TDOA has lower and upper bounds
- The source is “somewhere" onto a two-sheet hyberboloid