2. Methodological Foundations

1. Robot heads and acoustic laboratories
2. Binaural Processing Pipeline
3. Continuous-time Fourier transform
4. Continuous short-time Fourier transform
5. Discrete-time signals
6. Discrete short-time Fourier transform
7. Spectrogram of an acoustic signal
8. **Cross-correlation**
9. Relative transfer function
10. Binaural features

Cross-Correlation of Two Signals

- The cross-correlation (CC) function is a fundamental tool for comparing two signals:

$$r_{x_1,x_2}(\tau) = \sum_{t=1}^{T} x_1(t)x_2(t-\tau)$$

- Cross-correlation is a fundamental tool in binaural hearing as it allows to estimate the time difference between two signals recorded with two microphones:

$$\hat{\tau} = \underset{\tau}{\operatorname{argmax}}\, r_{x_1,x_2}(\tau)$$

Normalized Cross-Correlation

- The normalized cross-correlation (NCC) function varies between 0 (fully uncorrelated signals) and 1 (identical signals):

$$\rho_{x_1,x_2}(\tau) = \frac{r_{x_1,x_2}(\tau)}{\left(r_{x_1,x_1}(0)\, r_{x_2,x_2}(0)\right)^{1/2}}$$

- $r_{x,x}(0) = \sum_{t=1}^{T} |x(t)|^2$ is also called the power of the signal.

The Power Spectral Density

- In the spectral domain, one can compute the signal power for each frequency.
- First, we compute the power over a frame l

$$\phi_{x,x}(f,l) = X(f,l)X^*(f,l) = |X(f,l)|^2$$

- Second we compute the power over several frames:

$$\Phi_{x,x}(f) = \frac{1}{L}\sum_{l=1}^{L} \phi_{x,x}(f,l)$$

The Cross-Power Spectral Density

- We can also define the **cross-power** of two signals for a frame l:

$$\phi_{x_1,x_2}(f,l) = X_1(f,l)X_2^*(f,l),$$

- as well as for several frames:

$$\Phi_{x_1,x_2}(f) = \frac{1}{L}\sum_{l=1}^{L}\phi_{x_1,x_2}(f,l)$$

Session Summary

- Correlating two signals
- Power spectral density
- Cross-correlation in the spectral domain