Code-Based Cryptography

Other Cryptographic Constructions Relying on Coding Theory

Matthieu Finiasz

informatiques mathématiques Inria

Code-Based Cryptography

1. Error-Correcting Codes and Cryptography
2. McEliece Cryptosystem
3. Message Attacks (ISD)
4. Key Attacks
5. **Other Cryptographic Constructions Relying on Coding Theory**

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- **Code-Based Digital Signatures**
- The Courtois-Finiasz-Sendrier (CFS) Construction
- Attacks against the CFS Scheme
- Parallel-CFS
- Stern’s Zero-Knowledge Identification Scheme
- An Efficient Provably Secure One-Way Function
- The Fast Syndrome-Based (FSB) Hash Function

Digital Signatures

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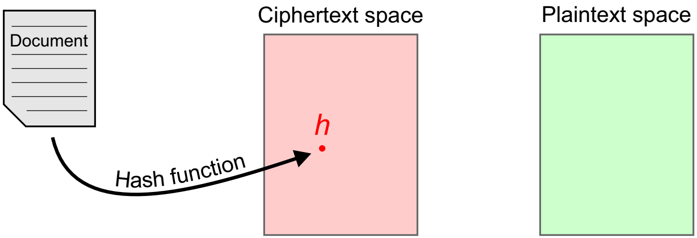
This is the “opposite” of the encryption operation in a public key scheme where:

- anyone can encrypt
- only one person can decrypt the resulting ciphertext

→ Digital signatures often use a decryption operation.

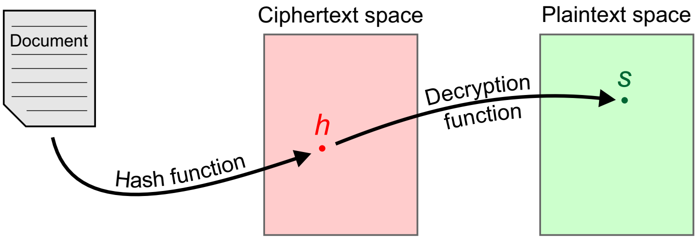
Implementing a Digital Signature



The signer hashes the document into an element of the ciphertext space using a public cryptographic hash function:

- allows to sign documents of arbitrary length
- ties the hash/ciphertext h to the document

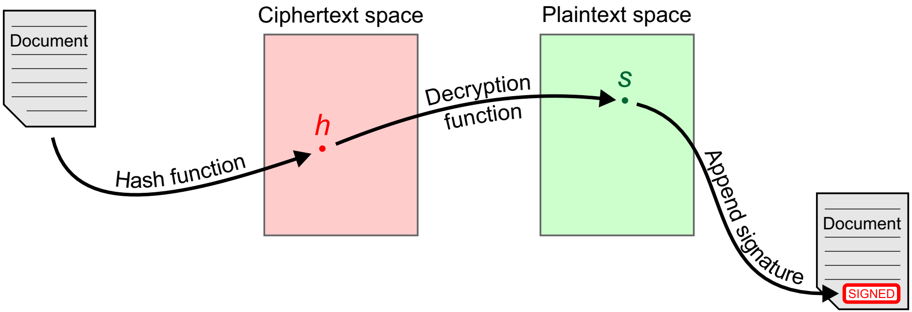
Implementing a Digital Signature



Then, the signer decrypts *h* into a plaintext *s*.

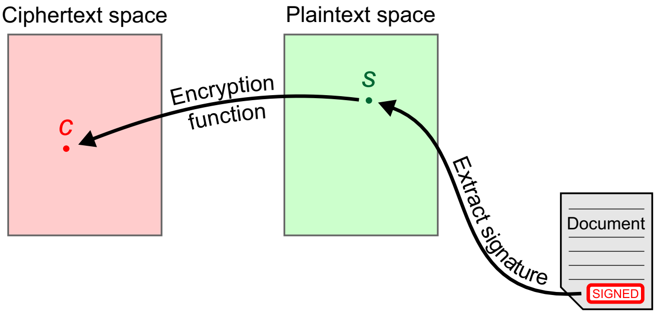
- requires the knowledge of the signer's secret key
- the plaintext *s* is the signature

Implementing a Digital Signature



The signer simply appends the signature to the document.

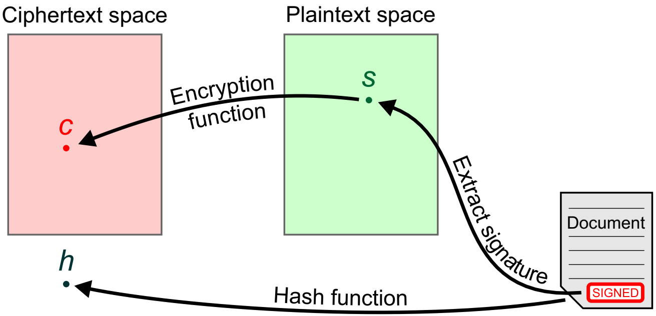
Implementing a Digital Signature



The verifier starts by extracting the signature *s* and re-encrypting it into a ciphertext *c*:

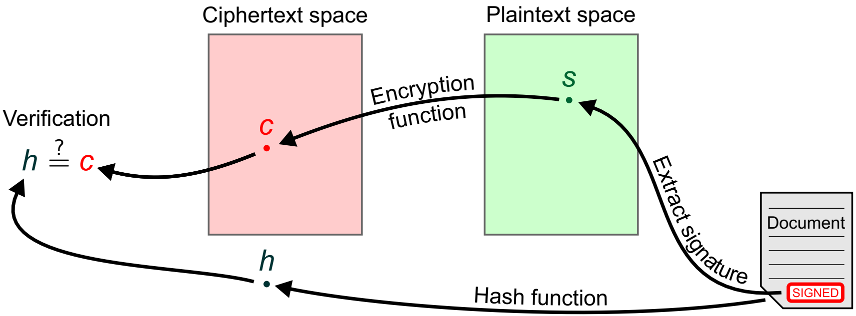
- only the public key of the signer is needed here
- encryption must be deterministic

Implementing a Digital Signature



The verifier also re-computes the hash h of the document.

Implementing a Digital Signature



The signature is considered valid if both the hash h and the ciphertext c are equal.

McEliece or Niederreiter?

The McEliece scheme:

- converts the plaintext into a message and encodes it
- adds some random errors to it

Problems:

- encryption is not deterministic
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The Niederreiter scheme:

- embeds the plaintext into an error pattern
- computes its syndrome

$\rightarrow$ encryption is deterministic

The Niederreiter scheme is used!

The Problem with Code-Based Cryptosystems

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For example, for RSA signatures:

- the ciphertext space is $[1, N-1]$
- any integer in this interval is a valid ciphertext

  $\rightarrow$ one simply needs to hash onto a uniformly distributed integer range

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  → deciding if a word/syndrome is decodable is hard
- hashing onto valid ciphertexts is impossible

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