HOCHSCHULE KONSTANZ
UNIVERSITY OF APPLIED SCIENCES

ENSICAEN
ÉCOLE NATIONALE SUPÉRIEURE D'INGÉNIEURS DE CAEN
& CENTRE DE RECHERCHE

CDMA Technology :

Pr. Dr. W. Skupin
www.htwg-konstanz.de

Pr. S. Flament
www.greyc.fr/user/99

CTI Commission des Titres d'Ingénieur
Normandie Université
CONFÉRENCE DES GRANDES ÉCOLES
EUR-ACE

CDMA Technology :

- Introduction to Spread Spectrum Technology
- CDMA / DS : Principle of operation
- Generation of PN Spreading Codes

- **Advanced Spreading Codes**

- Principles of CDMA/DS decoding
- Radio Cells & System Capacity
- Basics of Global Navigation Satellite Systems
- Galileo : European GNSS

CDMA Technology :

Advanced Spreading Codes

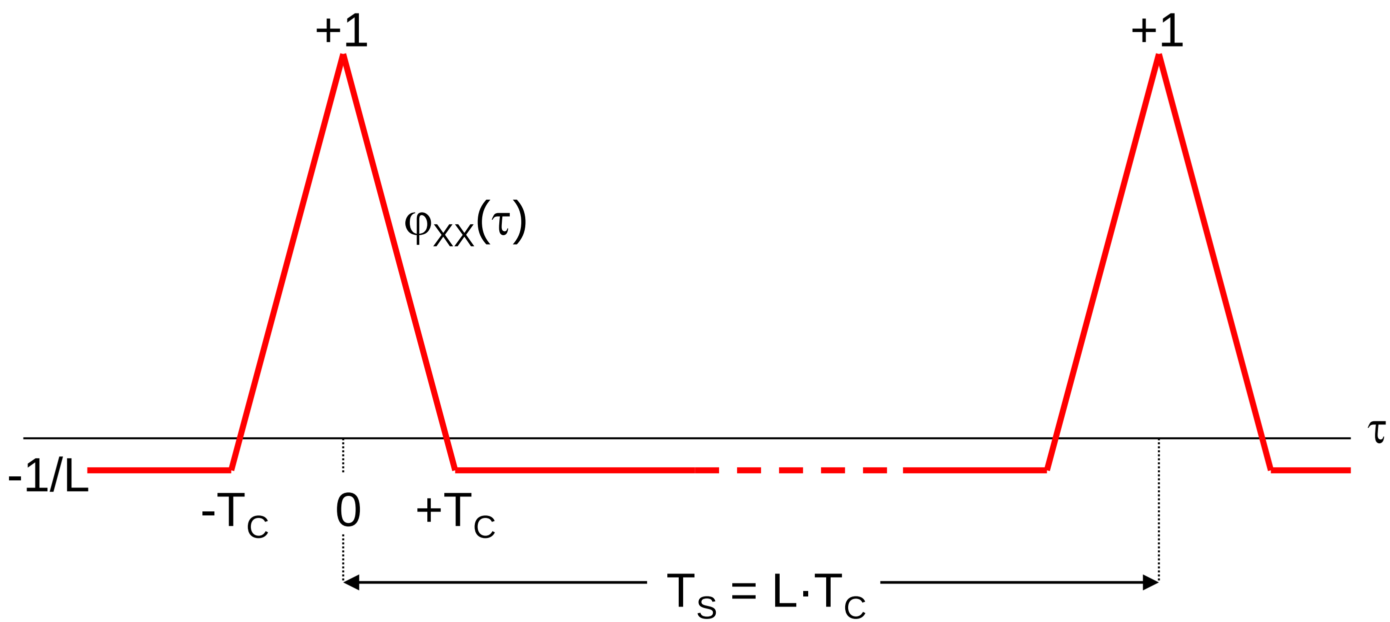
- Part 1 : fundamentals of m-sequences
- Part 2 : Gold Codes
- Part 3 : Walsh Functions
- Part 4 : Barker and Kasami Codes
- Part 5 : Choice and Applications of Code Families

Ideal spreading codes should offer following characteristics:

- noise like properties (i.e. equal probabilities for 0 and 1 ; no internal statistical dependencies ; uniform power density spectrum $\approx$ "white noise" ; etc.)
- good autocorrelation properties $\rightarrow$ no or low side peaks in the autocorrel. function ACF $\Rightarrow$ unambiguous correlation maximum for reliable receiver synchronisation
- codes have to be reproducable $\rightarrow$ tx and rx require absolutely identical codes for successful regeneration of the data signal (bandwidth compression)
- code multiplexing (CDMA) requires sets of othogonal equal length codes $\equiv$ "code families" of uncorrelated codes with equal code length for separation of different traffic channels

$\Rightarrow$ pseudo-noise codes (pn-codes) deliver a good approximation to these requirements with m-sequences being the most prominent representatives

ACF of a m-sequence: normalized representation!



Important results:

- ACF is periodical with $L \cdot T_C$ (T_C = clock/chip period)
- $\varphi_{XX}(\tau) = -1/L \approx 0$ for $|\tau| \geq T_C$ (uncorrelated for usual L)
- echoes with delay of $\tau \geq T_C$ are practically uncorrelated
- envelope of power density function (PDF) $\sim \sin^2(\pi T_C f) / (\pi T_C f)^2$

- For CDMA code multiplex sets of multiple orthogonal (uncorrelated) codes are required. In many applications the no. of m-sequences of the same degree n is too small!

- Gold codes can be easily generated from m-sequences and offer numerous families of orthogonal codes of equal code length (sequence length) L.

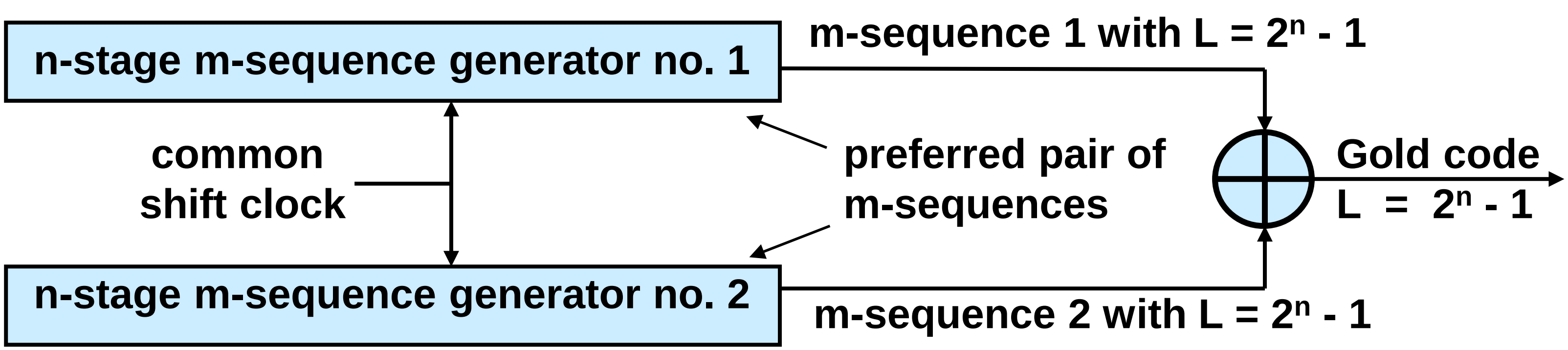
- Gold codes are (usually) generated by modulo-2-addition of 2 m-sequences with equal degree n. The selection of a preferred pair of m-sequences has to follow rules given by R. Gold (1967). These rules and their application require understanding of Galois fields (theory of numbers) and will not be presented in this lecture. However, examples for preferred pairs will be given later in this chapter and in following chapters in the context of Gold code applications.

- In contrast to m-sequences Gold codes show side lobes in their ACF. Properly chosen Gold codes have a guaranteed minimum peak-to-side lobe ratio in the ACF. Accordingly the maximum of the cross correlation function between different codes of the same family is definitely limited.

- Gold codes are widely used in spread spectrum and CDMA systems, e.g. for GPS and for UMTS.

- **Generation of Gold-Codes:**
  - 2 m-sequence generators with equal no. of stages
  - simultaneous clocking of generator 1 and generator 2
  - start condition of gen. 1: at least one "1" required in the register (no "all-0-state")
  - start condition of gen. 2: choice of $2^n - 1$ different start constellations (phases)

- **Block Diagram:**



- <u>No. of Gold codes per code family:</u>

  each phase shift between LFSR 1 and LFSR 2 produces an individual Gold code of length $L = 2^n - 1$

  $\rightarrow$ both m-sequence generators produce their individual m-sequence

  $\rightarrow$ there are $L = 2^n - 1$ possible phase shifts between 2 n-stage m-sequence generators

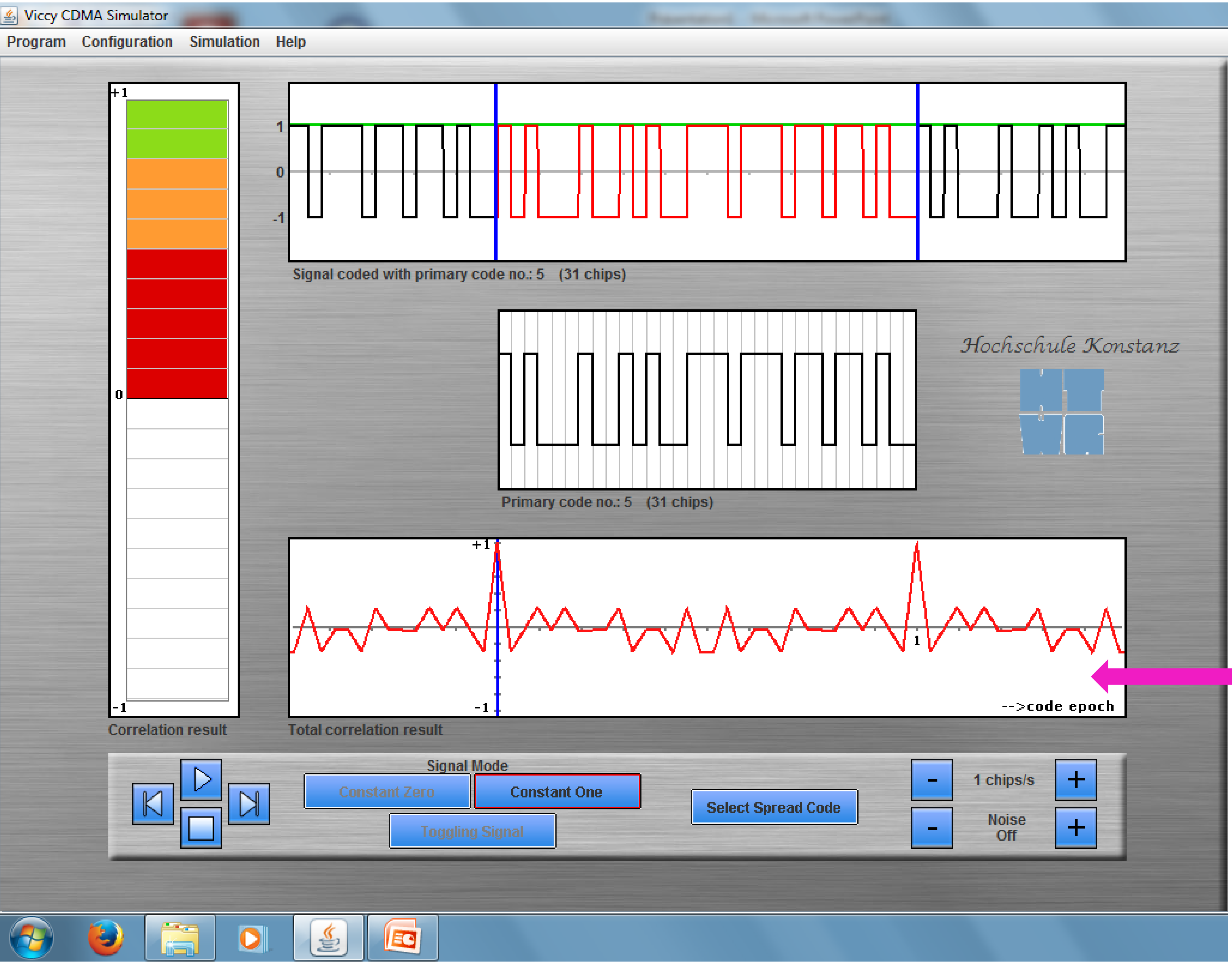
  $\Rightarrow$ a Gold code family, composed of 2 preferred m-sequences each of degree n, contains in total $2^n + 1$ different codes!

  example: $n = 5 \quad \rightarrow \quad 2^5 + 1 = 33$ codes per family

- <u>Attention:</u> no Gold codes of this restricted type are defined for $n = 4$ oder $n = i \cdot 4$

- **the ACF side peaks of Gold codes as well as the cross correlation between Gold codes of the same code family show 3 discrete levels (for properly chosen m-sequences!)**
- **properties of 3-level CCF of 2 Gold codes (formulas and examples):**

LFSR stages n	code length	CCF values (normalized)	probabilities of occurence
n odd	$L = 2^n - 1$	$-1/L$ $-[2^{(n+1)/2} + 1] / L$ $+[2^{(n+1)/2} - 1] / L$	≈ 0.50 ≈ 0.25 ≈ 0.25
n even (but not divisible by 4)	$L = 2^n - 1$	$-1/L$ $-[2^{(n+2)/2} + 1] / L$ $+[2^{(n+2)/2} - 1] / L$	≈ 0.750 ≈ 0.125 ≈ 0.125
example for: n=5 (odd)	L = 31	-1/31 = -0.032 -9/31 = -0.290 +7/31 = +0.226	≈ 0.50 ≈ 0.25 ≈ 0.25
example: n=10 (even and not divisible by 4)	L = 1023	-1/1023 = -0.00098 -65/1023 = -0.0635 +63/1023 = 0.0616	≈ 0.750 ≈ 0.125 ≈ 0.125

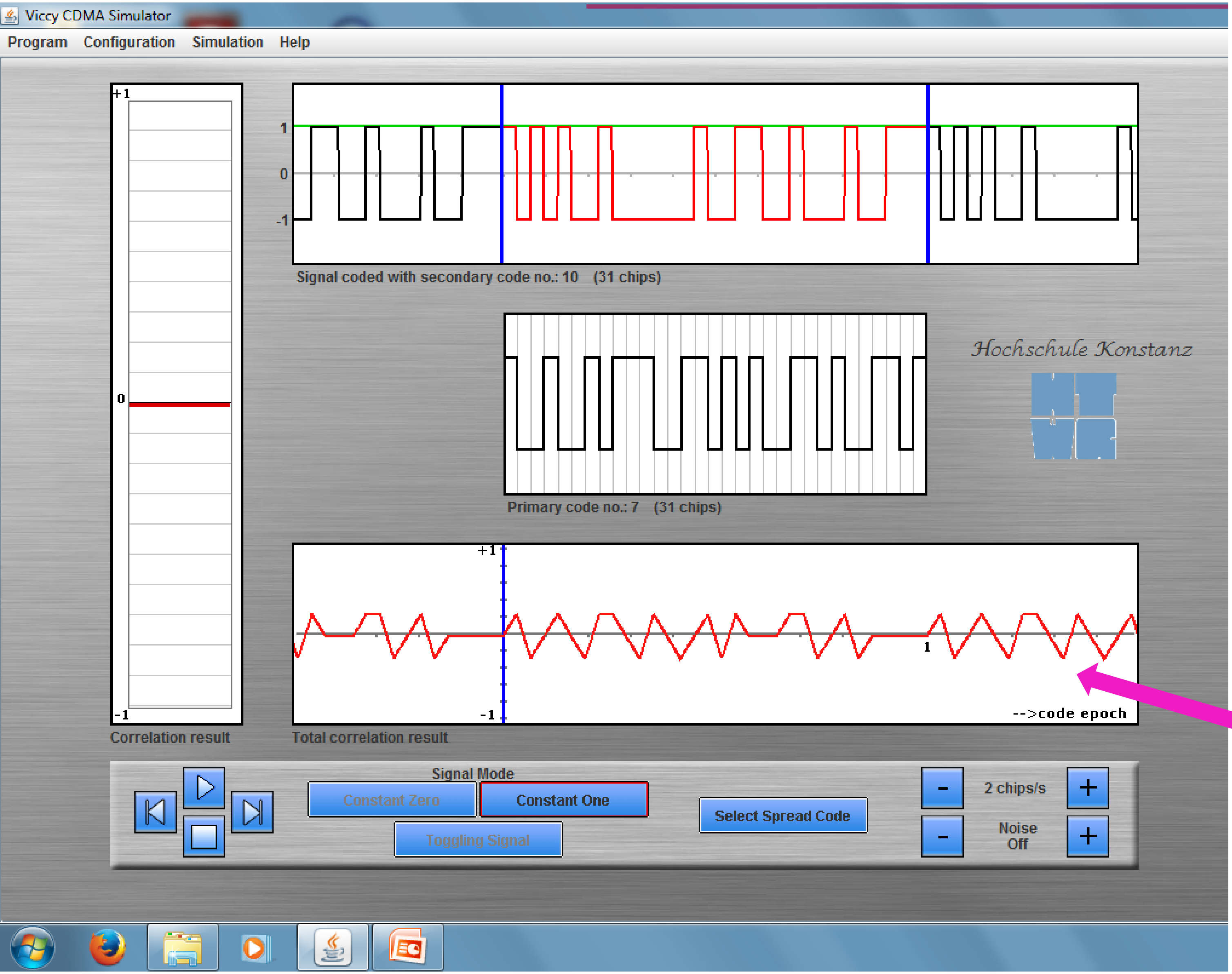


screen shot from proprietary CDMA simulation soft-ware Viccy

autocorrelation function of a Gold code of length 31 (primary code no. 5 selected from code family of 33 codes including the 2 generating m-sequences)

for this code: n=5 (odd) → 3 levels (besides acf max.) in the acf can be seen clearly

Part 2 : Gold Code CCF from Viccy Simulation



screen shot from proprietary CDMA simulation soft-ware Viccy

crosscorrelation function of 2 Gold codes of length 31 (primary code no. 7 and secondary code no. 10 from code family of 33 codes including the 2 generating m-sequences)

for thess codes: n=5 (odd) → 3 levels in the ccf can be seen clearly

Selected pairs of m-sequences for Gold code generation:

None of the procedures to find the preferred pairs of m-sequences for the generation of proper Gold codes has been described in this chapter. These procedures are based on calculation with "Galois Fields" and associated "mean root powers". In the context of this lecture the deduction of these procedures would exceed the intended level of details. However, some characteristic selections shall be given here because of their relevance for CDMA/DS demonstration and applications:

<u>Example 1:</u> Degree n = 5 $\Rightarrow$ 1. pair of preferred polynomials is: 45_{oct} and 67_{oct}
(this pair is used for "Viccy" simulation tool)
$\Rightarrow$ 2. pair of preferred polynomials is: 75_{oct} and 51_{oct}

<u>Example 2:</u> Degree n = 10 $\Rightarrow$ pair of preferred polynomials: 2011_{oct} and 3515_{oct}
this pair is used for GPS C/A-code generation (see chapter on GNSS)!

<u>Example 3:</u> Degree n = 18 $\Rightarrow$ pair of preferred polynomials:
$100\ 0201_{oct}$ and $100\ 2241_{oct}$
this pair is used for UMTS scrambling code generation

Deterioration of ACF and CCF properties with data modulated spreading codes

- m-sequences and Gold codes show no or only very little side peaks in the autocorrelation function (ACF) or small peaks in the crosscorrelation function (CCF). However, this is only true as long as these codes are not modulated with data bits. For data modulated spreading codes the ACF and the CCF contain unfavourable peaks due to partial correlation.

- The occurrence of these undesired side peaks can lead to problems with the selection of appropriate threshold levels for sync detect and DLL tracking! When higher threshold levels have to be selected the margin between "true" and "pseudo" correlation is reduced and will lead to increased bit error rates due to synchronization errors.

- The effect of partial correlation is particularly relevant for short spreading codes with low sequence lengths L. With increasing sequence length L or higher order n of the generator polynomial(s) this effect will be of less importance and the side peak levels in the ACF and the CCF will go down!

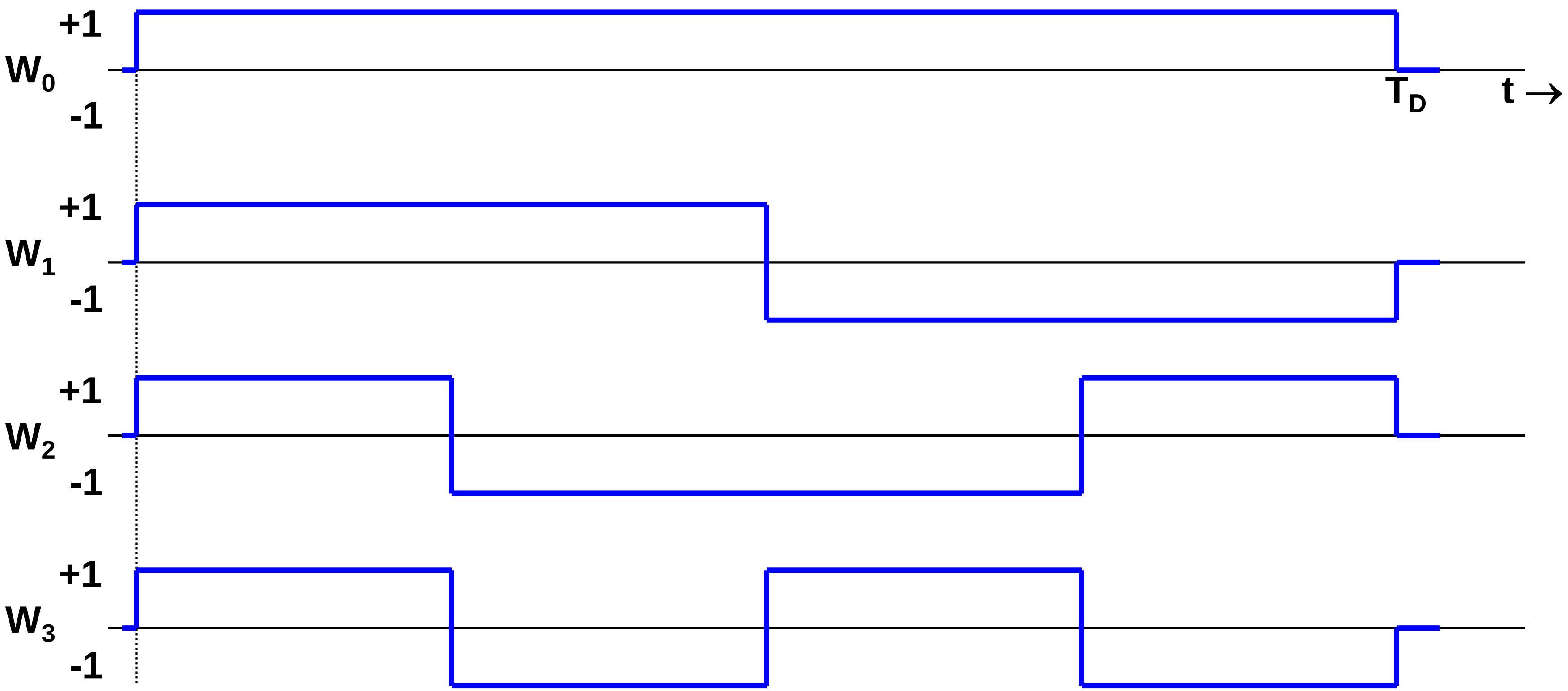
- 2 signals x(t) and y(t) are called "orthogonal", if the following condition is satisfied:

$$\int_{-\infty}^{+\infty} x(t) \cdot y(t)\, dt = 0 \qquad (\rightarrow \text{uncorrelated signals!})$$

- Multiplication of data signals (data bit duration: $T_D = T_{Data\ Bit}$) with spreading code signals in the basband is - in a general view - a baseband modulation with the spreading code as (baseband) carrier

- For CDMA code multiplexing multiple orthogonal carriers (codes) are required for traffic channel separation

- A well known family of orthogonal carriers are the time limited "Walsh functions" (some examples will be shown below)



The Walsh functions shown above are all orthogonal.
The generation of Walsh functions will be discussed in the following view graphs.

♦ Definition of Walsh Functions:

- Walsh functions are composed from the value sets {-1; +1} or {0; 1} respectively
- Walsh funktions of order N form a set of N time functions denoted:

$$\{ W_j(t) \; ; \; t \in (0 \; , \; T_D) \; ; \; j = 0, 1, 2, \; \ldots \; , N\text{-}1\}$$

♦ Properties of Walsh functions:

- $W_j(0) = 1$ for all j
- $W_j(t)$ has precisely j sign changes (zero crossings) in the interval $(0 \ldots T_D)$
- $$\int_0^{T_D} W_j(t)\, W_k(t)\, dt = \begin{cases} 0 & \text{if } j \neq k \\ T_D & \text{if } j = k \end{cases}$$ all codes are mutually orthogonal

♦ Generation of Walsh functions:

- Walsh functions are generated from so called "Hadamard matrices"

- Walsh functions are the rows of a Hadamard matrix.
- Hadamard matrices are generated by the following recursion:

$$H_1 = 0 \quad \text{and} \quad H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & \overline{H_N} \end{bmatrix}$$

- Examples for Hadamard matrices:

$$H_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$$H_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

- The rows of a N x N Hadamard matrix form the so called "Hadamard codes" in the order H_0 , H_1 , H_2 , ... , H_{N-1} .

- These rows have to be classified by their frequency of sign changes (zero crossings).

- The Walsh functions are the the rows of the Hadamard matrix in order of their sign changes: W_0 , W_1 , W_2 , ... , W_{N-1}
(the index correponds to the no. of sign changes!)

$\Rightarrow$ **Hadamard codes and Walsh functions do not differ in content but are differently indexed only!**

Example for Hadamard codes and Walsh functions of order 8:

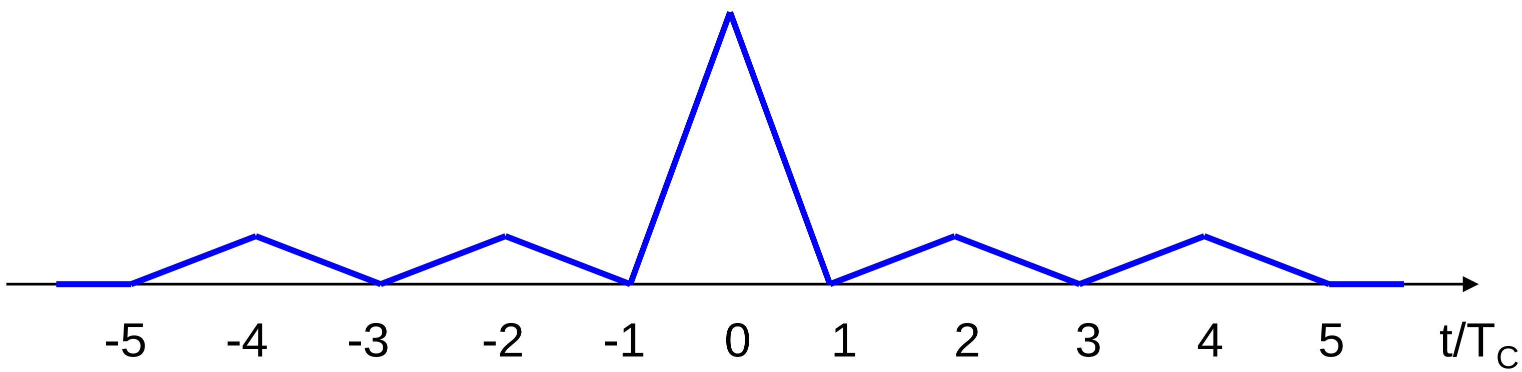
Hadamard code no.	bit sequence (rows of the Hadamard matrix)	no. of sign changes (zero crossings)	Walsh function no.
H_0	0 0 0 0 0 0 0 0	0	W_0
H_1	0 1 0 1 0 1 0 1	7	W_7
H_2	0 0 1 1 0 0 1 1	3	W_3
H_3	0 1 1 0 0 1 1 0	4	W_4
H_4	0 0 0 0 1 1 1 1	1	W_1
H_5	0 1 0 1 1 0 1 0	6	W_6
H_6	0 0 1 1 1 1 0 0	2	W_2
H_7	0 1 1 0 1 0 0 1	5	W_5

Note: the Walsh functions W_0 bis W_3 have already been shown in a previous viewgraph (however, in an inverted representation!)

- **Binary Barker codes offer favourable autocorrelation behaviour and ACF properties**
- **Unfortunately, there is only a very limited number of different Barker codes!**
- **Binary Barker codes consist of a finite number of binary symbols {+1 ; -1}**
- **Barker codes are defined by their (normalized) autocorrelation function ACF:**

$$\mathbf{ACF_{Barker}} = \begin{cases} 1 & \text{for } \tau = 0 \\ 0\ ;\ +1/N\ ;\ \text{or}\ -1/N & \text{for } \tau = \pm T_C\ ;\ \pm 2T_C\ ;\ \ldots\ ;\ \pm(N-1)T_C \\ 0 & \text{for } \tau = \geq NT_C \text{ und } \tau = \leq -NT_C \end{cases}$$

- **<u>Example:</u> normalized ACF for Barker code with N = 5 :**



- Notwithstanding intensive research and computer aided search only 8 binary Barker codes have been found! As 2 of them are of the same order, Barker codes of only 7 orders exist!

- List of all known binary Barker codes:

N	Code
2	+ +
3	+ + -
4	+ + + - or + + - +
5	+ + + - +
7	+ + + - - + -
11	+ + + - - - + - - + -
13	+ + + + + - - + + - + - +

Note:
Symbols "+" and "–" represent bipolar (voltage) levels different from zero level! Outside the defined Barker code sequence the level is strictly zero! To be precise a binary Barker code consists of 2 levels $\neq 0$ and will show the level 0 beyond the edges of the code sequence!

- Application of binary Barker codes:
  - signal acquisition (detection of binary signal strings in the presence of noise/interference)
  - synchronization of data packets or preambles of data sequences
  - spreading of data bits in CDMA/DS systems (e.g. IEEE 802.11 WLAN early standards)

- Kasami code families are constructed from a maximum-length-sequence (m-sequ.)
- in contrast to Gold codes the Kasami code families have less members for a given sequence length L, but offer better side peak properties in acf and ccf
- a Kasami code family is derived from a m-sequence with <u>even</u> degree n and hence has a sequence length of: $L = 2^n - 1$
- this m-sequence will be decimated by a decimation factor d with:

$$d = 2^{n/2} + 1$$

This results in a "support" sequence (s-sequence) with a sequence length of:

$$L_S = 2^{n/2} - 1$$

- in addition to the basic m-sequence the other members of a Kasami code family are constructed by multiplication of the m-sequence with all L_S cyclically shifted versions of the support sequence (s-sequence)
- the number of codes in a (so called "small") Kasami code family - including the basic m-sequence - is:

$$M = 2^{n/2}$$

Part 4 : Example for Kasami Code Family

Attention: all codes are in bipolar format! For logic representation replace: $-1 \equiv$ "0"; $+1 \equiv$ "1"

A Kasami code family shall be developed from a given basic m-sequence with the following properties:
- degree: $n = 4$
- bit sequ.: +1 +1 +1 +1 –1 –1 –1 +1 –1 –1 +1 +1 –1 +1 –1

- decimation factor: $d = 2^2 + 1 = 5$
- no. of codes in family: $M = 2^2 = 4 \Rightarrow$ 3 additional codes have to be constructed!
- support sequence: decimation of m-sequence by $d = 5$ (every 5th element of the m-sequence has to be selected) $\Rightarrow$ s-sequence: +1 –1 +1
- construction of $M-1 = 3$ additional Kasami codes:

m-sequence $\otimes$ {+1 –1 +1} = +1 –1 +1 +1 +1 –1 –1 –1 –1 –1 –1 +1 –1 –1 –1

m-sequence $\otimes$ {–1 +1 +1} = –1 +1 +1 –1 –1 –1 +1 +1 –1 +1 +1 +1 +1 +1 –1

m-sequence $\otimes$ {+1 +1 –1} = +1 +1 –1 +1 –1 +1 –1 +1 +1 –1 +1 –1 –1 +1 +1

- different code families offer different characteristics, benefits and weaknesses for CDMA.

- m-sequences offer the best approximation to noise signals and have no side lobes in their ACF. Unfortunately, the no. of m-sequences of equal length L is very low.

- Gold codes offer a sufficient no. of codes of equal length L, but their ACF shows side peaks (≡ partial correlation)!
  However, the maxima of these side peaks are strictly limited!

- Kasami codes offer better ACF characteristics (smaller side peaks), but compared to Gold codes the code families have less members.

- Walsh functions are perfectly orthogonal, but their ACFs are rather unequal.
  So the power density spectra of Walsh functions differ considerably thus requiring different bandwidth.

- Binary Barker codes have a clear and single ACF maximum.
  For $|\tau| \geq N \cdot T_C$ the ACF is 0 (totally uncorrelated).
  Unfortunately, there are only very few Barker codes!

- Barker codes are best suited for code acquisition and clock synchronization of short data bursts.

- m-sequences, Gold codes, and Kasami codes are well suited for "scrambling" or spreading of signals because of their well defined bandwidth and the sufficient number of codes

- Walsh functions are typ. used for channel separation (separating data streams) because of their ideal orthogonality. Usually Walsh functions are used in combination with Gold codes (e.g. channel separation for scrambled signals)

Required Process Gain

The required process gain will directly determine the spreading factor. For high Multipath immunity and/or robust communication longer codes, which result in higher spreading factors, have to be selected. However, it has to be guaranteed that the required bandwidth is available. Eventually the data bit rate has to be reduced to match the bandwidth requirements.

Required Number of Codes

The required number of different traffic channels directly defines the number of required codes. This determines which type of code will be applicable. Codes with big code families usually offer less desirable ACF and CCF properties than codes with only few members in a family. If possible m-sequences and/or Kasami codes should be preferred to Gold codes. However, real world systems mostly require code families which can be realized best with Gold codes.

Future Development

The development of optimum spreading codes is a never ending story. For special purposes and applications research on optimized codes is still going on. Additionally aspects may occur for interoperability between different systems (e.g. GPS & Galileo). This will have effects on the code structure and the preferred type of code .

- **Different Spreading Codes are used for Different Applications**
  - **Maximum-length sequences offer good statistical behaviour**
  - **Gold Codes allow for bigger code families of the same degree n**
  - **Gold codes must be constructed from preferred pairs of m-sequences**
  - **Walsh functions are strictly orthogonal and applicable for channel separation**
  - **Kasami codes are a good compromise between m-sequences and Gold codes**
  - **Barker codes are well suited for signal acquisition, but not for spreading**

- **Key points :**
  - **m-sequences offer optimum autocorrelation function (acf)**
  - **Gold codes require preferred pairs of m-sequences for code generation**
  - **Walsh functions offer strictly orthogonal (channelization) codes**
  - **other codes exist (Kasami, Barker, etc.) and will be used for special applications**

- **If needed, a few related readings in Spreading Codes :**

  - **Holmes, Jack K. :** **Spread Spectrum Systems**
    Artech House Inc., Norwood, MA 2007

  - **Peterson, Wesley W.; :** **Error Correcting Codes (Appendix C)**
    MIT Press ; seventh printing 1984
    ISBN: 0-262-16039-0